DETERMINISTIC MIMO CHANNEL ORDER ESTIMATION BASED ON CANONICAL CORRELATION ANALYSIS

Marta Arroyo, Javier Vía, and Ignacio Santamaría

Dept. of Communications Engineering, University of Cantabria, Spain
E-mail: {martaay, jvia, nacho}@gtas.dicom.unican.es
Telephone: +34-942201552. Fax: +34-942201488

ABSTRACT

Channel order estimation is a critical step in blind channel identification/equalization algorithms. In this paper, a new criterion for channel order estimation of multiple-input multiple-output (MIMO) channels is presented. The proposed method relies on the reformulation of the blind equalization problem as a set of nested canonical correlation analysis (CCA) problems, whose solutions are given by a generalized eigenvalue (GEV) problem. In particular, the channel order estimates are obtained from the multiplicity of the largest generalized eigenvalue of the successive GEVs. Unlike previous approaches, the performance of the proposed method is good even in the cases of small data sets, colored signals, and channels with small head and tails terms, which is illustrated by means os some numerical examples.

1. INTRODUCTION

Recently, blind identification and equalization of multiple-input multiple-output (MIMO) systems have received a lot of attention due to its capability to extract the channel, or the sources, without the penalty in bandwith efficiency associated to the transmission of a training sequence. Blind channel equalization methods can be divided into two groups. On the one hand, stochastic methods are based on the estimate of the correlation matrices of the observations and typically make some assumptions about the input. On the other hand, the deterministic techniques are solely based on the subspace decomposition of the received data matrices, which translates into a better performance when only a few observations are available. However, deterministic techniques are usually based on the previous knowledge of the channel order. Addressing this issue, several methods have been proposed for the particular case of single-input multiple-output (SIMO) systems (see [1], [2], [3] and the references therein). Unfortunately, the number of channel order estimation techniques for general MIMO systems is rather scarce [4].

In this work we present a new deterministic technique for the estimation of the MIMO channel order, which is based on the reformulation of the blind equalization problem as a set of nested canonical correlation analysis (CCA) problems [5]. The CCA solutions are given by a generalized eigenvalue problem (GEV). Specifically, the equalizers are obtained from the eigenvector associated to the largest eigenvalue, whose multiplicity is directly related to the MIMO channel orders. Based on this fact, we propose to analyze the multiplicities for different channel orders, which provides the sufficient information to exactly recover the channel orders in the noiseless case. In order to deal with more realistic scenarios, we introduce a denoising step, which allows us to obtain an estimate of the theoretical eigenvalues. Finally, these estimates are compared with a threshold, obtaining the estimated multiplicities and channel orders.

This paper is organized as follows. In Section 2 we review the formulation of the blind equalization problem. Section 3 introduces the proposed method of deterministic MIMO channel order estimation, whose performance is illustrated in Section 4 by means of some numerical examples. Finally, the main conclusions of this work are presented in Section 5.

2. BLIND EQUALIZATION OF MIMO CHANNELS

2.1. Notation and Data Model

We consider the noise free MIMO system shown in Fig. 1 where signals $\mathbf{x}[n] = [x_1[n], ..., x_P[n]]^T$, are the outputs of an unknown M-input/P-output finite impulse response (FIR) system driven by M signals, $s_1[n], \ldots, s_M[n]$.

Stacking K successive observation vectors into $\tilde{\mathbf{x}}[n] = \left[\mathbf{x}^T[n], ..., \mathbf{x}^T[n-K+1]\right]^T$, we obtain

$$\tilde{\mathbf{x}}[n] = \sum_{i=1}^{M} \mathcal{T}(\mathbf{h}_i)\tilde{\mathbf{s}}_i[n], \tag{1}$$

This work was supported by the Spanish Government (MEC) under project TEC2007-68020-C04-02.

978-1-4244-2241-8/08/$25.00 ©2008 IEEE

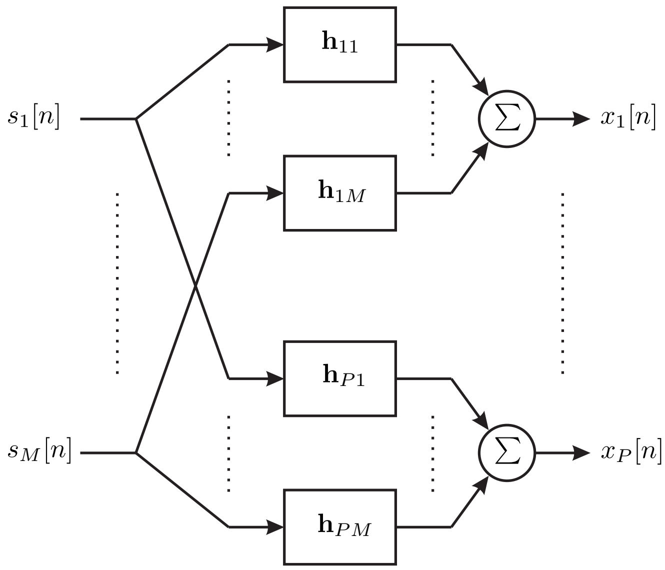

Fig. 1. Multiple-Input Multiple-Output (MIMO) system.

where $\tilde{\mathbf{s}}_i[n] = [s_i[n], ..., s_i[n-K-L_i+2]]^T$, $\mathbf{h}_i$ denotes the FIR-SIMO channel of length L_i associated to the *i*-th source signal, and

$$\mathcal{T}(\mathbf{h}_i) = \begin{bmatrix} \mathbf{h}_i[0] & \cdots & \mathbf{h}_i[L_i-1] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}_i[0] & \cdots & \mathbf{h}_i[L_i-1] \end{bmatrix},$$

is the $PK \times (K+L_i-1)$ filtering matrix associated to the *i*-th FIR-SIMO channel. Eq.(1) can be expressed in a more compact form as

$$\tilde{\mathbf{x}}[n] = \mathcal{T}(\mathbf{H})\tilde{\mathbf{s}}[n],$$

where $\tilde{\mathbf{s}}[n] = \left[\tilde{\mathbf{s}}_1^T[n], ..., \tilde{\mathbf{s}}_M^T[n]\right]^T$, and $\mathcal{T}(\mathbf{H}) = [\mathcal{T}(\mathbf{h}_1) \cdots \mathcal{T}(\mathbf{h}_M)]$ is the MIMO channel filtering matrix.

2.2. Blind MIMO Equalization Based on Channel Order Diversity

Recently, in [5] the authors have proposed a technique for blind equalization based on channel order diversity. If $\mathcal{T}(\mathbf{H})$ is full-column rank, it can be easily proved that there exists a set of M matrices $\mathbf{W}_i$ of size $PK \times Q_i$, such that

$$[\mathbf{W}_1 \cdots \mathbf{W}_M]^T \mathcal{T}(\mathbf{H}) = \mathbf{W}^T \mathcal{T}(\mathbf{H}) = \mathbf{I},$$

where $Q_i = K + L_i - 1$ is the number of equalizers that allows us to extract the delayed versions of the *i*-th source, which is distorted by a SIMO channel of length L_i. Denoting the *k*-th column of $\mathbf{W}_i$ as $\mathbf{w}_{ik}$, we can write, for $i = 1, ..., M$ and $k = 1, ..., Q_i$

$$\mathbf{w}_{ik}^T \tilde{\mathbf{x}}[n+k-1] = s_i[n],$$

i.e., the columns of the left-inverse of $\mathcal{T}(\mathbf{H})$ provide a set of zero-forcing (ZF) equalizers with differents delays for the M source signals. This implies, for $i = 1, ..., M$,

$$\mathbf{w}_{ik}^T \tilde{\mathbf{x}}[n+k] = \mathbf{w}_{il}^T \tilde{\mathbf{x}}[n+l], \quad k, l = 1, ..., Q_i,$$

which constitutes the main idea of the blind equalization method proposed in [5]. Specifically, defining the following matrix from a block of $Q_1 + N - 1$ observation vectors,

$$\tilde{\mathbf{X}}_k[n] = \underbrace{[\tilde{\mathbf{x}}[n+k-1] \quad \cdots \quad \tilde{\mathbf{x}}[n+k+N-2]]^T}_{N \times PK},$$

with $k = 1, ..., Q_i$, the blind equalization procedure can be rewritten as the following optimization problem

$$\underset{\hat{\mathbf{w}}_{i1}, \ldots, \hat{\mathbf{w}}_{iQ_i}}{\arg\min} \sum_{k,l=1}^{Q_i} \left\| \tilde{\mathbf{X}}_k[n]\hat{\mathbf{w}}_{ik} - \tilde{\mathbf{X}}_l[n]\hat{\mathbf{w}}_{il} \right\|^2, \qquad (2)$$

subject to the following constraint on the energy of the output signals

$$\sum_{k=1}^{Q_i} \left\| \tilde{\mathbf{X}}_k \hat{\mathbf{w}}_{ik} \right\|^2 = 1.$$

Interestingly, the above problem is the maximum variance (MAXVAR) generalization of canonical correlation analysis (CCA) to several data sets ($\tilde{\mathbf{X}}_1[n], ..., \tilde{\mathbf{X}}_{Q_i}[n]$) [6], whose solutions are given by the following generalized eigenvalue (GEV) problem [7]

$$\frac{1}{Q_i} \tilde{\mathbf{R}}_i \hat{\mathbf{w}}_i = \beta_i \tilde{\mathbf{D}}_i \hat{\mathbf{w}}_i, \qquad (3)$$

where $\hat{\mathbf{w}}_i = \left[\hat{\mathbf{w}}_{i1}^T, ..., \hat{\mathbf{w}}_{iQ_i}^T\right]^T$, $\hat{\mathbf{w}}_{ik}$ are the canonical vectors, the matrices $\tilde{\mathbf{R}}_i$ and $\tilde{\mathbf{D}}_i$ are defined as

$$\tilde{\mathbf{R}}_i = \begin{bmatrix} \tilde{\mathbf{R}}_{11} & \cdots & \tilde{\mathbf{R}}_{1Q_i} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{R}}_{Q_i1} & \cdots & \tilde{\mathbf{R}}_{Q_iQ_i} \end{bmatrix},$$

$$\tilde{\mathbf{D}}_i = \begin{bmatrix} \tilde{\mathbf{R}}_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \tilde{\mathbf{R}}_{Q_iQ_i} \end{bmatrix},$$

and $\tilde{\mathbf{R}}_{kl} = \tilde{\mathbf{X}}_k^H[n]\tilde{\mathbf{X}}_l[n]$ are the estimates of the crosscorrelation matrices. Thus, after extracting the equalizers $\hat{\mathbf{w}}_i$, the signal estimate is obtained as the best linear combination of the equalizer outputs which is given by

$$\hat{\mathbf{s}}_i[n] = [\hat{s}_i[n], \ldots, \hat{s}_i[n+N-1]]^T = \frac{1}{Q_i} \sum_{k=1}^{Q_i} \tilde{\mathbf{X}}_k[n]\hat{\mathbf{w}}_{ik}.$$

Finally, we must point out that in the case of rank-deficient matrices $\mathcal{T}(\mathbf{H})$, the channel can be easily decomposed into identifiable (full-column rank) and non-identifiable parts. Thus, the techniques presented in this paper can be applied to the extraction of the equalizers, or channel orders, associated to the identifiable part of the channel.

3. MIMO ORDER ESTIMATION BASED ON CCA

The blind equalization method summarized in the previous subsection is used here to derive a new technique for the estimation of the MIMO channel orders. Let us start by discussing the properties of the CCA-based blind equalization method.

3.1. Preliminaries

In [5] it has been proven that the theoretical solutions of the CCA problem in (3) are the following:

- **Desired solutions**: Obviously, the blind equalization criterion is satisfied by the equalizers extracting the signals distorted by a SIMO channel of length L_i.
- ***Spurious* solutions**: It is easy to prove that the equalization criterion is also satisfied by the set of equalizers extracting the sources distorted by longer SIMO channels ($L_j > L_i$), and their delayed versions. In particular, if

$$\hat{\mathbf{w}}_j = \left[\hat{\mathbf{w}}_{j1}^T, \ldots, \hat{\mathbf{w}}_{jQ_j}^T\right]^T,$$

are the equalizers of the *j*-th source signal, then

$$\hat{\mathbf{w}}_i = \left[\hat{\mathbf{w}}_{jk}^T, \ldots, \hat{\mathbf{w}}_{j(k+Q_i-1)}^T\right]^T,$$

for $k = 1, \ldots, L_j - L_i + 1$ also satisfy the blind equalization criterion.

Therefore, assuming that there exists a subset of θ_i FIR-SIMO channels of length L_i, the theoretical solutions of the proposed blind equalization criterion span a subspace of dimension $\theta_i + \theta_i^{\perp}$, where

$$\theta_i^{\perp} = \sum_{L_j > L_i} (L_j - L_i + 1)$$

represents the number of signals, and their delayed replicas, affected by SIMO channels of length $L_j > L_i$. Finally, in the absence of noise this is translated into a multiplicity of order $\theta_i + \theta_i^{\perp}$ of the largest eigenvalue of the GEV in (3).

3.2. Proposed Technique

The multiplicity of the generalized eigenvalues is exploited in this subsection to propose a new channel order estimation technique. Let us assume that the conditions associated to the blind equalization technique in [5] are satisfied for an upper bound $\hat{L}_{max}$ of the MIMO channel order. Then, solving the problem in (3) for each $\hat{L}_{max} \geq L \geq 1$, we obtain that the theoretical multiplicity of the largest generalized eigenvalue satisfies

$$mult(\max(\beta_i)) = \theta_i + \theta_i^{\perp}, \quad i = 1, ..., M.$$

Algorithm 3.1 Summary of the CCA-MAXVAR technique for MIMO channel order estimation.

Select $\hat{L}_{max}$ and K.
Initialize $L = \hat{L}_{max}$ and $i = 1$
while $L > 1$ **do**
  Calculate $Q = K + L - 1$.
  Form the CCA-MAXVAR problem with data sets $\tilde{\mathbf{X}}_1[n], ..., \tilde{\mathbf{X}}_Q[n]$.
  Obtain the multiplicity of the largest generalized eigenvalue, $mult(\max(\beta))$
  Obtain the number of constraints to apply $\theta^{\perp} = \sum_{\hat{L}_j \geq L} (\hat{L}_j - L + 1)$.
  Calculate $\theta = mult(\max(\beta)) - \theta^{\perp}$.
  if $\theta > 0$ **then**
    We can conclude than there exist θ sources distorted by SIMO channels of length L_i.
  end if
  Update $i = i + \theta$.
  Update $L = L - 1$.
end while

Thus, the proposed channel order estimation technique, which is summarized in Algorithm 3.1, reduces to the analysis of these multiplicities.

In a practical situation the observations are corrupted by noise, and the blind equalization criterion can not be exactly satisfied, which translates into different generalized eigenvalues (canonical correlations) $\beta_i < 1$. In order to determine the multiplicities we need to introduce two additional stages into the proposed technique. Firstly, the effect of the noise is compensated by means of a soft denoising step. Specifically, the eigenvalues are corrected by means of the following formula

$$\beta_i = \hat{\beta}_i \frac{\hat{\mathbf{w}}_i^H \tilde{\mathbf{D}}_i \hat{\mathbf{w}}_i}{\hat{\mathbf{w}}_i^H (\tilde{\mathbf{D}}_i - \hat{\sigma}^2 \mathbf{I}) \hat{\mathbf{w}}_i} - \frac{1}{Q_i} \frac{\hat{\mathbf{w}}_i^H \hat{\mathbf{R}}_{n_i} \hat{\mathbf{w}}_i}{\hat{\mathbf{w}}_i^H (\tilde{\mathbf{D}}_i - \hat{\sigma}^2 \mathbf{I}) \hat{\mathbf{w}}_i},$$

which provides a measure of the residual inter-symbol interference (ISI) at the output of the equalizers, and where $\hat{\mathbf{R}}_{n_i}$ is a matrix which models the effect of the white noise, of variance $\hat{\sigma}^2$, on $\tilde{\mathbf{R}}_i$. Finally, the multiplicities are estimated by comparing the corrected eigenvalues with a given threshold, i.e., all the eigenvalues above the threshold are considered as *maximum* eigenvalues, and therefore they provide the multiplicities and the channel orders.

Finally, we must point out that the proposed technique has been derived from a deterministic framework. Therefore, in the absence of noise it is able to exactly recover the channel orders within a finite number of observations. On the other hand, in the presence of noise the proposed method provides very promising results, which is illustrated by means of some numerical examples in the following section.

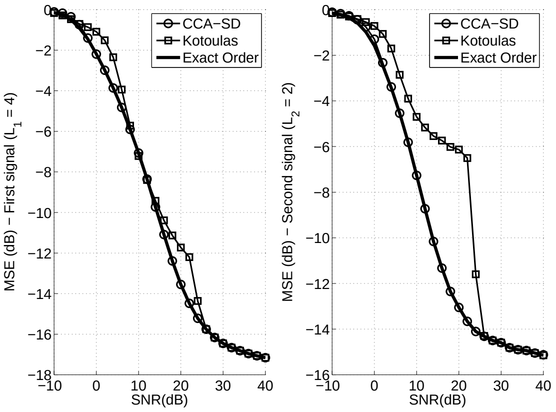

Fig. 2. Final MSE after equalization.

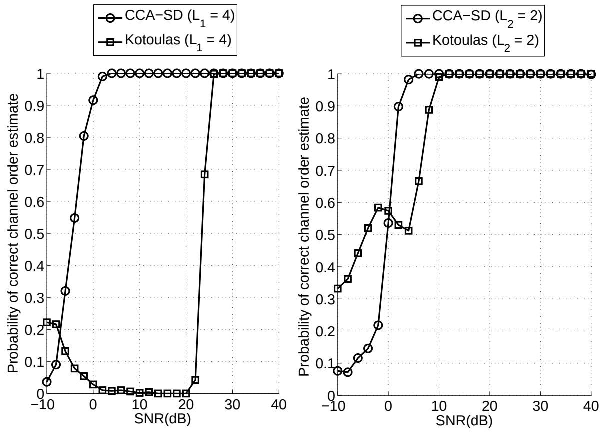

Fig. 3. Probability of correct channel order detection.

4. SIMULATION RESULTS

In this section the performance of the proposed technique is evaluated by means of several simulation examples. The results were obtained as the average of 500 independent realizations of a single experiment. We consider unit-variance 16-QAM source signals distorted by a MIMO channel and corrupted by zero-mean Gaussian noise. The MIMO channel is the same used in the experiments of [5], which corresponds with the propagation model of Clarke [8]. In particular, we consider a 2×4 MIMO channel ($M = 2$, $P = 4$) with effective[1] SIMO channel lengths $L_1 = 4$ and $L_2 = 2$. Finally, in all the examples we consider $N = 500$ observations, and the proposed technique is compared with the method in [4] (denoted here as Kotoulas).

Channel order estimation is the first step in any blind identification/equalization technique, where the final goal is to restore the original source signal. We have used the channel order estimate to derive a set of equalizers using the algorithm described in Section 2. In the first example two white sources are distorted by the 2×4 MIMO channel, which includes small heading and trailing terms. The final mean squared error (MSE) after equalization is shown in Fig. 2, where we can see that the best results are provided by the proposed method. Fig. 3 shows the probability of correct channel order detection as a function of the SNR. Here we can clearly see that the technique based on CCA outperforms Kotoulas method for both SIMO channels. Finally, in Fig. 4 we observe the probability density function (estimated with the Parzen windowing method [9]) of the channel order estimates for SNR = 10 dB. As can be seen, the proposed technique recovers the correct channel orders, whereas the method proposed by Kotoulas underestimates the channel order of the longest SIMO channel.

In the second example, the sources have been colored by a FIR filter with impulse response $\mathbf{b} = [1, 1]$. The final MSE after channel order detection and equalization is shown in Fig. 5, which illustrates the good performance of the proposed technique even in the presence of colored sources.

[1] The effective channel length is considered as the number of taps that concentrate most of the channel energy [1].

5. CONCLUSIONS

In this paper, a new multiple-input multiple-output (MIMO) channel order estimation technique has been proposed. The method is based on a previously proposed blind equalization technique, which can be reformulated as a set of nested canonical correlation analysis (CCA) problems. In particular, in the absence of noise the channel order estimates are directly given by the multiplicities of the largest eigenvalue of a generalized eigenvalue problem (GEV). Thus, in a realistic scenario the eigenvalues are firstly corrected by means of a denoising step, and the multiplicities are obtained as the number of eigenvalues above a fixed threshold. Finally, the performance of the proposed technique has been illustrated by means of numerical examples.

6. REFERENCES

[1] A. P. Liavas, P. A. Regalia, and J. P. Delmas, “Blind channel approximation: Effective channel order determination,” *IEEE Transactions on Signal Processing*, vol. 47, no. 12, pp. 3336–3344, 1999.

[2] L. Tong and Q. Zhao, “Joint order detection and blind channel estimation by least squares smoothing,” *IEEE Transactions on Signal Processing*, vol. 47, no. 9, pp. 2345–2355, Sept. 1999.

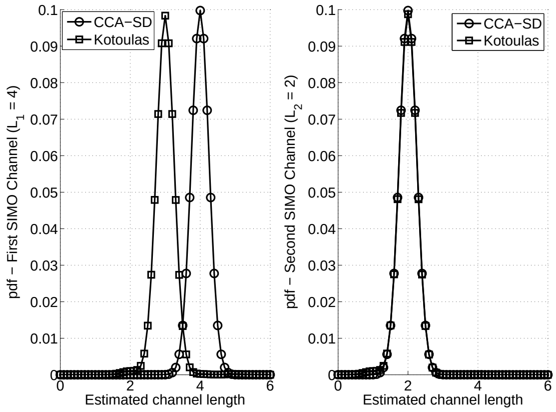

Fig. 4. Probability density functions (estimated using the Parzen method).

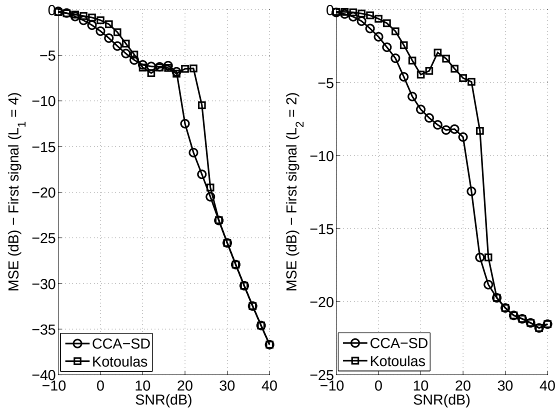

Fig. 5. Final MSE after equalization with colored inputs.

[3] J. Vía, I. Santamaría, and J. Pérez, "Effective channel order estimation based on combined identification/equalization," *IEEE Transactions on Signal Processing*, vol. 54, no. 9, pp. 3518–3526, Sept. 2006.

[4] D. Kotoulas, P. Koukoulas, and N. Kalouptsidis, "Subspace projection based blind channel order estimation of MIMO systems," *IEEE Transactions on Signal Processing*, vol. 54, no. 4, pp. 1351–1363, Apr. 2006.

[5] J. Vía, I. Santamaría, and J. Pérez, "Deterministic CCA-based algorithms for blind equalization of FIR-MIMO channels," *IEEE Transactions on Signal Processing*, vol. 55, no. 7, pp. 3867–3878, July 2007.

[6] J. R. Kettenring, "Canonical analysis of several sets of variables," *Biometrika*, vol. 58, no. 3, pp. 433–451, 1971.

[7] J. Vía, I. Santamaría, and J. Pérez, "Canonical correlation analysis (CCA) algorithms for multiple data sets: Application to blind SIMO equalization," in *European Signal Processing Conference (EUSIPCO 2005)*, Antalya, Turky, Sept. 2005.

[8] R.H. Clarke, "A statistical theory of mobile radio reception," *Bell Systems Technical Journal*, vol. 47, pp. 957–1000, 1968.

[9] E. Parzen, "On estimation of a probability density function and mode," *Annals of Mathematical Statistics*, vol. 33, pp. 1065–1076, 1962.